**Physics IV**
**ISI B.Math**
**Final Exam : April 30, 2015**

**Total Marks: 50**
**Time : 3 hours**
**Answer all questions**

1. (**Marks: 5 + 5 = 10** )

a) A particle of mass $m_1$ and velocity $\mathbf{v_1}$ collides with a stationary particle of mass $m_2$ and is absorbed by it. Find the mass $m$ and velocity $\mathbf{v}$ of the resultant compound system.

b) A and B travel to the right along the x-axis at speeds $\frac{4c}{5}$ and $\frac{3c}{5}$ with respect to the ground . C is situated between A and B and is also travelling to the right along the x-axis . How fast should C travel so that she sees A and B approaching her with equal speeds ? What is the speed of A and B as seen by C ?

2. (**Marks : 3 + 4 + 3 =10**)

(a) Show that the energy $E$ must exceed the minimum value of $V(x)$ for every normalizable solution to the time independent Schrödinger equation in one dimension.

(b) Let $P_{ab}(t)$ be the probability of finding a particle of mass $m$ in the range $(a < x < b)$ at time $t$. Show that

$$\frac{dP_{ab}}{dt} = J(a,t) - J(b,t)$$

where $J(x,t) = \frac{i\hbar}{2m}\left(\Psi(x,t)\frac{\partial \Psi^*(x,t)}{\partial x} - \Psi^*(x,t)\frac{\partial \Psi(x,t)}{\partial x}\right)$ and $\Psi(x,t)$ is the wave function of the particle

(c) If $V(x)$ is an even function of $x$, then the solutions $\psi(x)$ of the one-dimensional time-independent Schrödinger equation can always be taken to be either even or odd.

3. (**Marks : 4 + 2 + 4** )

A particle mass $m$ moves under the influence of the potential $V(x) = 0$ if $0 \leq x \leq a$ and $\infty$ otherwise.

(a) Solve the time independent Schrödinger equation for this potential and find the stationary states $\psi_n(x)$ and their corresponding energies $E_n$.

(b) If the initial state is given by $\Psi(x,0) = A[\psi_1(x) + \psi_2(x)]$, find $A$.

(c) Find $< x >$ in the state $\Psi(x,t)$ and show that it oscillates in time. What is the amplitude and angular frequency of the oscillation ?

4. (**Marks : 1 + 2 + 3 + 4 = 10** )

A one dimensional harmonic oscillator of mass $m$ has potential energy $V(x) = \frac{1}{2}m\omega^2 x^2$. Consider the operators $a = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega x + ip)$ and $a^\dagger = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega x - ip)$

(a) Do $a$ and $a^\dagger$ correspond to physically measurable observables ? Justify your answer.

(b) Express the Hamiltonian $\hat{H}$ in terms of $a$ and $a^\dagger$

(c) Show that if $\psi_n$ is a solution of the time independent Schrödinger equation with energy $E_n$, then $a\psi_n$ is a solution with energy $(E_n - \hbar\omega)$

(d) Given that $a^\dagger\psi_n = \sqrt{n+1}\psi_{n+1}$ and $a\psi_n = \sqrt{n}\psi_{n-1}$, find the expectation value of the potential energy $< V >$ in the state $\psi_n$. Remember that $\psi_n$s are orthonormal.

5. (**Marks : 2 + 3 + 2 + 2 + 1 = 10** )

(a) The Hamiltonian operator for a particle moving in a potential $V(x)$ is given by $\hat{H} = \frac{\hat{p}^2}{2m} + V(x)$ Compute $[H, \hat{p}]$ . Is it possible in general to simultaneously measure $\hat{H}$ and $\hat{p}$ to arbitrary accuracy ? Explain .

(b) A particle moving in one dimension has the wavefunction $\Psi(x,t) = A\exp[i(ax - bt)]$ , where $a$ and $b$ are constants. In what potential field $V(x)$ is the particle moving ? Compute the commutator in (a) in the case of the specific Hamiltonian for the case in (b). Is it possible to simultaneously measure energy and momentum with arbitrary accuracy in this case ?

(c) Referring to the wavefunction in (b), if the momentum of the particle is measured , what value is found? (in terms of $a$ and $b$?)

(d) If the energy is measured , what value is found?

(e) Will a measurement of position in this state yield a definite value ? Explain.